Chem. 540
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Angular Momentum – Problem 3

Consider the eigenstates of $\hat{S}_z$ for a spin $1/2$ particle. In this basis, any spin state is specified by two coefficients, i.e., by a two-component vector. As usual, the vector representation of the two eigenstates themselves is simply the two vectors with coefficients 1, 0 and 0, 1 as discussed in class. Thus, in this basis the matrix representation of the operator $\hat{S}_z$ is diagonal with elements $\frac{1}{2}\hbar$ and $-\frac{1}{2}\hbar$.

(a) Use the spin ladder operators to show that the matrix representation of $\hat{S}_x$ and $\hat{S}_y$ is given by $\frac{1}{2}\hbar$ times the Pauli spin matrices introduced in class.

(b) Calculate the commutators of the Pauli spin matrices.

(c) Show that the three Pauli spin matrices plus the 2×2 identity matrix form a complete set. It is sufficient to show that any 2×2 matrix can be expanded in terms of those.