32. $\hat{L}_+ = \hat{L}_x + i\hat{L}_y \qquad \hat{L}_- = \hat{L}_x - i\hat{L}_y$

a) $[\hat{L}_+, \hat{L}_z] = [\hat{L}_x + i\hat{L}_y, \hat{L}_z] = [\hat{L}_x, \hat{L}_z] + i[\hat{L}_y, \hat{L}_z]$

$= -i\hbar\hat{L}_y + i(i\hbar\hat{L}_x) = -i\hbar\hat{L}_y - \hbar\hat{L}_x$

$= -\hbar\hat{L}_+$

$[\hat{L}_+, \hat{L}_-] = [\hat{L}_x + i\hat{L}_y, \hat{L}_x - i\hat{L}_y]$

$= [\hat{L}_x, \hat{L}_x] - i[\hat{L}_x, \hat{L}_y] + i[\hat{L}_y, \hat{L}_x] + [\hat{L}_y, \hat{L}_y]$

$= 0 - i(i\hbar\hat{L}_z) + i(-i\hbar\hat{L}_z) + 0$

$= 2\hbar\hat{L}_z$

$[\hat{L}_+, \hat{L}^2] = [\hat{L}_x + i\hat{L}_y, \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2]$

$= [L_x, L_x^2] + [L_x, L_y^2] + [L_x, L_z^2] + i[L_y, L_x^2] + i[L_y, L_y^2] + i[L_y, L_z^2]$

$[L_x, L_y^2] = L_y[L_x, L_y] + [L_x, L_y]L_y$

$= L_y(i\hbar L_z) + (i\hbar L_z)L_y = i\hbar(L_yL_z + L_zL_y)$

$[L_x, L_z^2] = L_z[L_x, L_z] + [L_x, L_z]L_z$

$= L_z(-i\hbar L_y) + (-i\hbar L_y)L_z = -i\hbar(L_zL_y + L_yL_z)$

$[L_y, L_x^2] = L_x[L_y, L_x] + [L_y, L_x]L_x$

$= L_x(-i\hbar L_z) + (-i\hbar L_z)L_x = -i\hbar(L_xL_z + L_zL_x)$

$[L_y, L_z^2] = L_z[L_y, L_z] + [L_y, L_z]L_z$

$= L_z(i\hbar L_x) + (i\hbar L_x)L_z = i\hbar(L_zL_x + L_xL_z)$

$[\hat{L}_+, \hat{L}^2] = i\hbar(L_yL_z + L_zL_y - L_zL_y - L_yL_z) + i(i\hbar)(L_zL_x + L_xL_z - L_xL_z - L_zL_x)$

$[\hat{L}_+, \hat{L}^2] = 0$

b) $\hat{L}_x, \hat{L}_y, \hat{L}_z$ and $\hat{L}^2$ are hermitian.

$\hat{L}_+$ and $\hat{L}_-$ are not hermitian:

$(\hat{L}_+)^\dagger = L_x - iL_y = \hat{L}_-$

$(\hat{L}_-)^\dagger = \hat{L}_+$