33. Show that $|00\rangle$ and $|10\rangle$ eigenstates of angular momentum operators are orthogonal.

$\langle 00 | 10 \rangle = 0$ $\quad Y_{\ell,m}(\theta,\varphi) = \frac{(-1)^\ell}{2^\ell \ell!}\sqrt{\frac{2\ell+1}{4\pi}}\sqrt{\frac{(\ell+m)!}{(2\ell)!(\ell-m)!}}\, e^{im\varphi}\sin^{-m}\theta\left(\frac{d}{d\cos\theta}\right)^{\ell-m}\sin^{2\ell}\theta$

So: $Y_{00} = \frac{1}{\sqrt{4\pi}}$ $\quad Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta$

$$\langle 00|10\rangle = \int Y_{00}^* Y_{10}\, d\tau = \int_0^{2\pi}\int_0^{\pi} Y_{00}^* Y_{10} \sin\theta\, d\theta\, d\varphi$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{1}{\sqrt{4\pi}}\cdot\sqrt{\frac{3}{4\pi}}\cos\theta\sin\theta\, d\theta\, d\varphi$$

$$= \frac{\sqrt{3}}{4\pi}(2\pi)\int_0^{\pi}\cos\theta\sin\theta\, d\theta$$

$$= \frac{\sqrt{3}}{2}\int_{-1}^{1} u\, du \quad \text{where } u = \cos\theta$$

$$= \frac{\sqrt{3}}{2}\,\frac{1}{2}\,u^2\Big|_{-1}^{1} = \frac{\sqrt{3}}{4}(1-1) = 0$$

$\langle 00|10\rangle = 0$, therefore they are orthogonal.