35. $\Psi(\theta,\phi) = A\{Y_{1,+1}(\theta,\phi) + Y_{1,-1}(\theta,\phi)\}$.

a) $A = ?$

$\langle\Psi|\Psi\rangle = A^2\{\langle 1,1| + \langle 1,-1|\}\{|1,1\rangle + |1,-1\rangle\}$
$= A^2\{1 + 0 + 0 + 1\}$

$A = 1/\sqrt{2}$ to normalize $\langle\Psi|\Psi\rangle = 1$.

b) $L_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ $\quad L_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$

$\det\left\{\frac{\hbar}{2}\begin{pmatrix} -\lambda_x & 1 & 0 \\ 1 & -\lambda_x & 1 \\ 0 & 1 & -\lambda_x \end{pmatrix}\right\} = 0 \implies \lambda_x = 0 \text{ or } \pm\sqrt{2}$

The eigenvectors associated with these are:

$|\psi_x\rangle_0 = \frac{1}{\sqrt{2}}(|1,1\rangle - |1,-1\rangle)$ e.v. 0

$|\psi_x\rangle_+ = \frac{1}{\sqrt{2}}|1,0\rangle + \frac{1}{2}|1,1\rangle + \frac{1}{2}|1,-1\rangle$ e.v. $\hbar$

$|\psi_x\rangle_- = -\frac{1}{\sqrt{2}}|1,0\rangle + \frac{1}{2}|1,1\rangle + \frac{1}{2}|1,-1\rangle$ e.v. $-\hbar$

~~Therefore one half of me~~ $|\Psi\rangle = \frac{1}{\sqrt{2}}(|\psi_x\rangle_+ + |\psi_x\rangle_-)$

Therefore half the measurements yield $+\hbar$, half $-\hbar$.

For L_y: $\lambda_y = 0, \pm\sqrt{2}$

$|\psi_y\rangle_0 = \frac{1}{\sqrt{2}}(|1,1\rangle + |1,-1\rangle)$ e.v. 0.

$|\Psi\rangle = |\psi_y\rangle_0$, all measurements yield 0.

For L_z: the eigenvectors are $|1,1\rangle$ and $|1,-1\rangle$ so one half of measurements yield $+\hbar$ and the other half yield $-\hbar$.

L^2: all measurements yield $2\hbar^2$.

c) From individual measurements, we can determine the form of the wavefunction. The probability gives us $|c_n|^2$ from which we can determine the coefficients of the wavefunction. Just from the expectation values all we know is that it is in the $l=1$ state since $\langle L_x\rangle = \langle L_y\rangle = \langle L_z\rangle = 0$ and $\langle L^2\rangle = 2\hbar^2$.