Chem. 540
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## BASICS PROBLEM 5

Consider a particle moving on the *xy* plane. Suppose that the motion of the particle on the plane can be described by a Hamiltonian

$$\hat{H} = \hat{H}_1 + \hat{H}_2$$

where the operators $\hat{H}_1$ and $\hat{H}_2$ describe the motion of a particle in the *x* and in the *y* direction, respectively:

$$\hat{H}_1 = \frac{\hat{p}_x^2}{2m} + V_1(\hat{x}), \qquad \hat{H}_2 = \frac{\hat{p}_y^2}{2m} + V_2(\hat{y}) .$$

Suppose we call $\phi_n(x)$ the eigenfunctions of $\hat{H}_1$ with eigenvalues $\varepsilon_n$ and $\psi_k(y)$ the eigenfunctions of $\hat{H}_2$ with eigenvalues $\xi_k$. Show that the eigenfunction of the total Hamiltonian $\hat{H}$ are products of the type

$$\phi_n(x)\psi_k(y) .$$

Find the corresponding eigenvalues.

Whenever the Hamiltonian can be decomposed into a sum of operators involving different coordinates, we say that it is separable, and as you see its solution is trivial in those cases. A separable Hamiltonian means that the motion of the particle in each coordinate is independent of the motion in the other coordinates. This is not the case in general; any potential field that depends simultaneously on *x* and on *y* would make the *x* motion coupled to the *y* motion; in that case, the eigenstates would no longer be expressible in product form.