

Basics Problem 1 - Solution

$$(a) \int_{-\infty}^{\infty} dx |\psi_1(x)|^2 = \int_{-\infty}^{\infty} dx a^2 e^{-m\omega x^2/\hbar} = a^2 \sqrt{\frac{\pi\hbar}{m\omega}} = 1 \Rightarrow$$

$$a = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}$$

$$\int_{-\infty}^{\infty} dx |\psi_2(x)|^2 = \int_{-\infty}^{\infty} dx b^2 x^2 e^{-m\omega x^2/\hbar} = b^2 \frac{\sqrt{\pi}}{2} \left(\frac{\hbar}{m\omega}\right)^{3/2} = 1 \Rightarrow$$

$$b = \sqrt{2} \pi^{-1/4} \left(\frac{m\omega}{\hbar}\right)^{3/4}$$

Note on integral: $\int_{-\infty}^{\infty} dx e^{-Ax^2} x^2 = -\int_{-\infty}^{\infty} dx \frac{d}{dA} e^{-Ax^2} = -\frac{d}{dA} \sqrt{\frac{\pi}{A}}$

$$(b) \text{ Overlap: } \int_{-\infty}^{\infty} dx \psi_1^*(x) \psi_2(x) = ab \int_{-\infty}^{\infty} x e^{-m\omega x^2/\hbar} dx = 0$$

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 odd even