

Basics Problem 2 - Solution

$$\begin{aligned}
 (a) \quad \int_0^L dx \, a^2 \cos^2\left(\frac{\pi x}{L} - \frac{1}{2}\pi\right) &= \int_0^L a^2 \frac{1}{2} \left[1 + \cos\left(\frac{2\pi x}{L} - \pi\right)\right] dx \\
 &= \frac{a^2}{2} \int_0^L \left(1 - \cos \frac{2\pi x}{L}\right) dx = \frac{a^2}{2} \left(L - \frac{L}{2\pi} \sin \frac{2\pi x}{L} \Big|_0^L\right) \\
 &= \frac{a^2}{2} L = 1 \quad \Rightarrow \quad a = \sqrt{\frac{2}{L}}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad P_{\frac{1}{4}L, \frac{3}{4}L} &= \int_{L/4}^{3L/4} \frac{2}{L} \cos^2\left(\frac{\pi x}{L} - \frac{1}{2}\pi\right) dx = \frac{2}{L} \frac{1}{2} \int_{L/4}^{3L/4} \left(1 - \cos \frac{2\pi x}{L}\right) dx \\
 &= \frac{1}{L} \left[x - \frac{L}{2\pi} \sin \frac{2\pi x}{L} \right] \Big|_{L/4}^{3L/4} = \frac{1}{2} + \frac{1}{\pi}
 \end{aligned}$$