Chem. 540

Problem Basics 3 – Solution

(a) $$\int_0^L |\psi_2(x)|^2 dx = b^2 \int_0^L \cos^2\left(\frac{2\pi x}{L} - \frac{\pi}{2}\right) dx = \frac{b^2}{2} \int_0^L \left[1 + \cos\left(\frac{4\pi x}{L} - \pi\right)\right] dx$$

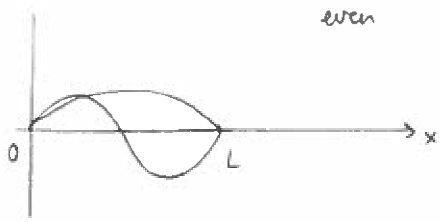
$$= \frac{b^2}{2}\left(L - \frac{L}{4\pi} \sin \frac{4\pi x}{L}\Big|_0^L\right) = \frac{b^2 L}{2} = 1 \Rightarrow b = \sqrt{\frac{2}{L}}$$

(b) $$\int_0^L |\psi(x)|^2 dx = N^2 \int_0^L \left[\psi_1(x)^2 + \psi_2(x)^2 + 2\psi_1(x)\psi_2(x)\right] dx$$

$$= N^2 \int_0^L \psi_1(x)^2 dx + N^2 \int_0^L \psi_2(x)^2 dx$$

$$+ 2N^2 \int_0^L \cos\left(\frac{\pi x}{L} - \frac{\pi}{2}\right) \cos\left(\frac{2\pi x}{L} - \frac{\pi}{2}\right) dx$$

$$= N^2 + N^2 + 2N^2 \int_0^L \underbrace{\sin \frac{\pi x}{L}}_{\text{even}} \underbrace{\sin \frac{2\pi x}{L}}_{\text{odd}} dx = 2N^2 = 1$$



$\Rightarrow \quad N = 1/\sqrt{2}$