

1. Probability of finding the electron at  $x$

$$P(x) = |\langle x|1\rangle\langle 1|s\rangle + \langle x|2\rangle\langle 2|s\rangle|^2$$

Write  $\langle x|j\rangle\langle j|s\rangle \equiv \rho_j e^{i\theta_j}$ ,  $j=1,2$

where  $\rho_j = |\langle x|j\rangle\langle j|s\rangle| = |\langle x|j\rangle| \cdot |\langle j|s\rangle|$ .

We know  $\frac{|\langle 2|s\rangle|^2}{|\langle 1|s\rangle|^2} = \frac{1}{100} \Rightarrow |\langle 2|s\rangle| = \frac{1}{10} |\langle 1|s\rangle|$

Since we are looking at a maximum and a minimum near the center and close to one another, we may assume  $|\langle x|2\rangle| \approx |\langle x|1\rangle|$ .

Then

$$\frac{\rho_2}{\rho_1} = \frac{|\langle x|2\rangle|}{|\langle x|1\rangle|} \cdot \frac{|\langle 2|s\rangle|}{|\langle 1|s\rangle|} = \frac{1}{10}$$

$$P(x) = \rho_1^2 + \rho_2^2 + 2\rho_1\rho_2 \cos(\theta_1 - \theta_2)$$

At a maximum,  $\cos(\theta_1 - \theta_2) = +1$  and  $P_{\max} = (\rho_1 + \rho_2)^2$

At a minimum,  $\cos(\theta_1 - \theta_2) = -1$  and  $P_{\min} = (\rho_1 - \rho_2)^2$

$$\frac{P_{\max} - P_{\min}}{P_{\min}} = \frac{4\rho_1\rho_2}{(\rho_1 - \rho_2)^2} = \frac{4\rho_2 \cdot 10\rho_2}{(9\rho_2)^2} = \frac{40}{81} \approx 0.5$$