Chem. 540
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PROBLEM FORMALISM 4

Suppose that $|\phi_1\rangle$ and $|\phi_2\rangle$ are two degenerate eigenstates of an operator $\hat{A}$, but they are not chosen to be orthogonal; however, they are normalized to unity. Let the overlap of these states be

$$\langle\phi_1|\phi_2\rangle = \lambda \text{ (real valued constant).}$$

a) Construct new mutually orthogonal states $|\Phi_1\rangle$ and $|\Phi_2\rangle$ that correspond to the same eigenvalue of $\hat{A}$. There are many ways to do this, but perhaps the simplest is to keep $|\Phi_1\rangle$ as is and replace $|\Phi_2\rangle$ by its component that is orthogonal to $|\Phi_1\rangle$. To do this, subtract from $|\Phi_2\rangle$ its component along $|\Phi_1\rangle$. (You may write the coefficient of this component as a variable and determine its value by requiring zero overlap with $|\Phi_1\rangle$.)

b) Are $|\phi_1\rangle$ and $|\phi_2\rangle$ orthogonal to other eigenstates $|\Phi_n\rangle$, $n > 2$ of $\hat{A}$ that have eigenvalues different from the eigenvalues of the two degenerate states? Are the new states $|\Phi_1\rangle$ and $|\Phi_2\rangle$ orthogonal to these other states?

c) Are the states $|\Phi_1\rangle$ and $|\Phi_2\rangle$ uniquely defined?