

$$78. [\hat{x}^2, \hat{p}_x] = \hat{x}^2 (-i\hbar \frac{\partial}{\partial x}) + i\hbar \frac{\partial}{\partial x} \hat{x}^2$$

$$[\hat{x}^2, \hat{p}_x] \psi = \hat{x}^2 (-i\hbar \frac{\partial \psi}{\partial x}) + i\hbar \frac{\partial}{\partial x} (x^2 \psi) = -i\hbar x^2 \frac{\partial \psi}{\partial x} + i\hbar (2x\psi + x^2 \frac{\partial \psi}{\partial x})$$

$$[\hat{x}^2, \hat{p}_x] \psi = 2i\hbar x \psi \Rightarrow [\hat{x}^2, \hat{p}_x] = 2i\hbar x$$

$$[\hat{x}, \hat{p}_x^2] = \hat{x} (-\hbar^2 \frac{\partial^2}{\partial x^2}) + \hbar^2 \frac{\partial^2}{\partial x^2} \hat{x}$$

$$[\hat{x}, \hat{p}_x^2] \psi = -\hbar^2 x \frac{\partial^2 \psi}{\partial x^2} + \hbar^2 \frac{\partial^2}{\partial x^2} (x\psi) = -\hbar^2 x \frac{\partial^2 \psi}{\partial x^2} + \hbar^2 (\frac{\partial}{\partial x} \psi + \frac{\partial}{\partial x} \psi + x \frac{\partial^2 \psi}{\partial x^2})$$

$$[\hat{x}, \hat{p}_x^2] \psi = 2\hbar^2 \frac{\partial \psi}{\partial x} \Rightarrow [\hat{x}, \hat{p}_x^2] = 2\hbar^2 \frac{\partial}{\partial x} = 2i\hbar \hat{p}_x$$

$$[\hat{x}^2, \hat{p}_y] = \hat{x}^2 (i\hbar \frac{\partial}{\partial y}) + i\hbar \frac{\partial}{\partial y} (x^2)$$

$$[\hat{x}^2, \hat{p}_y] \psi = \hat{x}^2 (-i\hbar \frac{\partial \psi}{\partial y}) + i\hbar \frac{\partial}{\partial y} (x^2 \psi) = -i\hbar x^2 \frac{\partial \psi}{\partial y} + x^2 i\hbar \frac{\partial \psi}{\partial y}$$

$$[\hat{x}^2, \hat{p}_y] = 0$$

$$[\hat{x}\hat{z}, \hat{p}_x] = \hat{x}\hat{z} (-i\hbar \frac{\partial}{\partial x}) + i\hbar \frac{\partial}{\partial x} \hat{x}\hat{z}$$

$$[\hat{x}\hat{z}, \hat{p}_x] \psi = xz (-i\hbar \frac{\partial \psi}{\partial x}) + i\hbar \frac{\partial}{\partial x} (xz\psi) = -i\hbar xz \frac{\partial \psi}{\partial x} + i\hbar z\psi + i\hbar xz \frac{\partial \psi}{\partial x}$$

$$[\hat{x}\hat{z}, \hat{p}_x] \psi = i\hbar z \psi \Rightarrow [\hat{x}\hat{z}, \hat{p}_x] = i\hbar \hat{z}$$