

$$17. \text{Tr } \hat{A} = \sum_n \langle n | \hat{A} | n \rangle$$

$$\text{Tr} (\hat{U}^\dagger \hat{A} \hat{U}) = \sum_n \langle n | \hat{U}^\dagger \hat{A} \hat{U} | n \rangle = \sum_n \sum_l \sum_m \langle n | \hat{U}^\dagger | l \rangle \langle l | \hat{A} | m \rangle \langle m | \hat{U} | n \rangle$$

$$= \sum_n \sum_l \sum_m \langle m | \hat{U} | n \rangle \langle n | \hat{U}^\dagger | l \rangle \langle l | \hat{A} | m \rangle$$

$$= \sum_l \sum_m \langle m | \hat{U} \hat{U}^\dagger | l \rangle \langle l | \hat{A} | m \rangle = \sum_l \sum_m \langle m | l \rangle \langle l | \hat{A} | m \rangle$$

$$= \sum_{l,m} \delta_{l,m} \langle l | \hat{A} | m \rangle = \sum_m \langle m | \hat{A} | m \rangle.$$

Since the trace of an operator is independent of the basis used,  $\text{Tr } \hat{A} = \text{Tr} (\hat{U}^\dagger \hat{A} \hat{U})$ .