$$H = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$\det \begin{pmatrix} 0-\lambda & -1 \\ -1 & 0-\lambda \end{pmatrix} = (-\lambda)^2 - 1 = \lambda^2 - 1 = 0 \qquad \lambda = \pm 1$$

$$(0-\lambda)\, c_1 - c_2 = 0$$

For $\lambda = 1$: $\quad -c_1 - c_2 = 0 \qquad c_2 = -c_1$

For $\lambda = -1$: $\quad +c_1 - c_2 = 0 \qquad c_2 = c_1$

So we have, after normalizing:

Eigenstate 1 (ground state): $|\psi_1\rangle = \frac{1}{\sqrt{2}}(|\phi_1\rangle + |\phi_2\rangle)$, $E_1 = -1$

Eigenstate 2 (excited state): $|\psi_2\rangle = \frac{1}{\sqrt{2}}(|\phi_1\rangle - |\phi_2\rangle)$, $E_2 = 1$

Spectral expansion: $\hat{H} = -|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|$

Matrix of eigenvectors: $\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \equiv U$

$$U \cdot U^{+} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$