20. $\int_{-\infty}^{\infty} dk\, e^{ikx} e^{-ak^2} = \int_{-\infty}^{\infty} e^{-ak^2+ikx}\, dk$

$$= \int_{-\infty}^{\infty} dk\, e^{-a\left(k-\frac{ix}{2a}\right)^2 - \frac{x^2}{4a}}$$

$$= e^{\frac{-x^2}{4a}} \int dk\, e^{-a\left(k-\frac{ix}{2a}\right)^2}$$

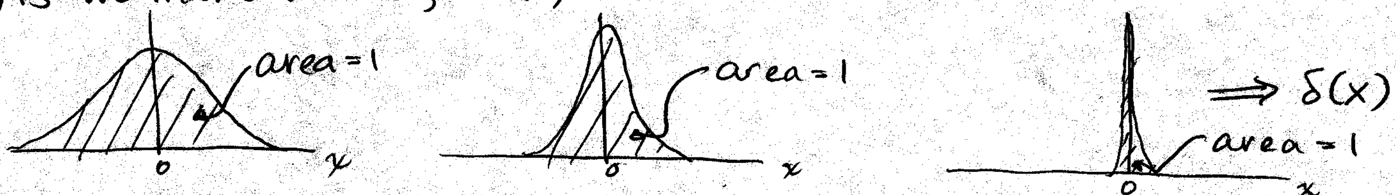
We know that $\int_{-\infty}^{\infty} dx\, e^{-ax^2} = \sqrt{\frac{\pi}{a}}$, so.

$$\int_{-\infty}^{\infty} dk\, e^{ikx} e^{-ak^2} = e^{-x^2/4a}\sqrt{\frac{\pi}{a}}$$

A gaussian function (normalized to one) is $g(x) = \frac{1}{\sqrt{\pi b^2}} e^{-x^2/b^2}$

$$\int_{-\infty}^{\infty} dk\, e^{ikx} e^{-ak^2} = \left(\frac{1}{\sqrt{4\pi a}} e^{-x^2/4a}\right)\left(\sqrt{4\pi a}\sqrt{\frac{\pi}{a}}\right) = 2\pi \cdot \left\{\frac{1}{\sqrt{4\pi a}} e^{-\frac{x^2}{4a}}\right\} = 2\pi \cdot G(x;a)$$

As we make $a \to 0$, $G(x;a)$ is



$$\lim_{a\to 0} \int_{-\infty}^{\infty} dk\, e^{ikx} e^{-ak^2} = \int_{-\infty}^{\infty} dk\, e^{0} e^{ikx} = \int_{-\infty}^{\infty} dk\, e^{ikx}$$

$$= \lim_{a\to 0} \left\{2\pi \frac{1}{\sqrt{4\pi a}} e^{-x^2/4a}\right\} = 2\pi\, \delta(x)$$

Since these are equal: $\int_{-\infty}^{\infty} dk\, e^{ikx} = 2\pi\, \delta(x)$