6 The momentum operator in one dimension is $\hat{p} = i\hbar \frac{\partial}{\partial x}$

An operator is hermitian if $\int dx\, \psi_1^*(x) \hat{A} \psi_2(x) = \left\{\int dx\, \psi_2^*(x) \hat{A} \psi_1(x)\right\}^*$

$$\int_{-\infty}^{\infty} dx\, \psi_1^*(x)\left(i\hbar \frac{\partial}{\partial x}\right)\psi_2(x) = \underbrace{i\hbar\, \psi_1^*(x)\psi_2(x)\Big|_{-\infty}^{\infty}}_{\to 0} - \int i\hbar \frac{\partial}{\partial x}\psi_1^*(x)\, \psi_2(x)\, dx \text{ (integrate by parts)}$$

$$\int dx\, \psi_1^*(x)\, i\hbar \frac{\partial}{\partial x} \psi_2(x) = \int dx\, \psi_2(x)\left(-i\hbar \frac{\partial}{\partial x}\right)\psi_1^*(x) = \left\{\int dx\, \psi_2^*(x)\left(i\hbar \frac{\partial}{\partial x}\right)\psi_1(x)\right\}^*$$

therefore $\hat{p}$ is a hermitian operator

The kinetic energy operator in one dimension is $\hat{K} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$.

$$\int dx\, \psi_1^*(x)\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right)\psi_2(x) = -\frac{\hbar^2}{2m}\psi_1^*(x)\frac{d}{dx}\psi_2(x)\Big|_{-\infty}^{\infty} - \int dx \left(\frac{\hbar^2}{2m}\right)\frac{d}{dx}\psi_2 \frac{d}{dx}\psi_1^*$$

$$= 0 - \left\{\frac{-\hbar^2}{2m}\frac{\partial}{\partial x}\psi_1^*(x)\psi_2(x) - \int dx\left(\frac{-\hbar^2}{2m}\right)\psi_2(x)\frac{d^2}{dx^2}\psi_1^*(x)\right\}$$

$$\int dx\, \psi_1^*(x)\left(\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\right)\psi_2(x) = \int dx\, \psi_2(x)\left(\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\right)\psi_1^*(x)$$

$$= \left\{\int dx\, \psi_2^*(x)\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right)\psi_1(x)\right\}^*.$$

The kinetic energy operator is hermitian and the product of hermitian operators is also a hermitian operator, if and only if the operators commute.