

$$12. \hat{A}_0 |\varphi_n\rangle = a_n |\varphi_n\rangle$$

If we shift the operator \hat{A}_0 by a constant, λ : $\hat{A}^\# = \hat{A}_0 + \lambda$:

$$\hat{A} |\varphi_n\rangle = (\hat{A}_0 + \lambda \hat{I}) |\varphi_n\rangle$$

$$= \hat{A}_0 |\varphi_n\rangle + \lambda \hat{I} |\varphi_n\rangle$$

$$= a_n |\varphi_n\rangle + \lambda |\varphi_n\rangle$$

$$\hat{A} |\varphi_n\rangle = (a_n + \lambda) |\varphi_n\rangle$$

Therefore the eigenfunctions $|\varphi_n\rangle$ remain the same, but the eigenvalues are shifted by the constant λ .