

$$E_k = \alpha + 2\beta \cos \frac{2k\pi}{N} \quad N \text{ even}, \quad k=0, \pm 1, \dots, \frac{N}{2} \quad N \text{ odd}, \quad k=0, \pm 1, \dots, \pm \frac{N-1}{2}$$

(a) Benzene: $N=6$

$$k=0 \quad E = \alpha + 2\beta$$

$$\pm 1 \quad \alpha + 2\beta \cos \frac{2\pi}{6} = \alpha + \beta \quad (\text{2-fold degeneracy})$$

$$\pm 2 \quad \alpha - \beta \quad (\text{2-fold degeneracy})$$

$$3 \quad \alpha - 2\beta$$

$$\text{LUMO} \quad \underline{\underline{\quad}} \quad \alpha - 2\beta$$

$$\text{Binding energy: } 2(\alpha + 2\beta) + 4(\alpha + \beta) = 6\alpha + 8\beta$$

$$\text{LUMO} \quad \underline{\quad} \quad \alpha - \beta$$

$$\text{Comparing to } 6(\alpha + \beta),$$

$$\text{HOMO} \quad \cancel{\underline{\quad}} \quad \alpha + \beta$$

$$\text{delocalization energy is } 2\beta.$$

$$\text{HOMO} \quad \cancel{\underline{\quad}} \quad \alpha + 2\beta$$

Cyclooctatetraene: $N=8$

$$k=0 \quad E = \alpha + 2\beta$$

$$\pm 1 \quad \alpha + 2\beta \cos \frac{2\pi}{8} = \alpha + \sqrt{2}\beta \quad (\text{2-fold degeneracy})$$

$$\pm 2 \quad \alpha + 2\beta \cos \frac{4\pi}{8} = \alpha \quad " \quad "$$

$$\pm 3 \quad \alpha + 2\beta \cos \frac{6\pi}{8} = \alpha - \sqrt{2}\beta \quad " \quad "$$

$$4 \quad \alpha - 2\beta$$

$$\text{LUMO} \quad \underline{\underline{\quad}} \quad \alpha - 2\beta$$

$$\text{Binding energy: } 2(\alpha + 2\beta) + 4(\alpha + \sqrt{2}\beta) + 2\alpha$$

$$\text{HOMO} \quad \cancel{\underline{\quad}} \quad \alpha$$

$$= 8\alpha + 4(1 + \sqrt{2})\beta$$

$$\text{HOMO} \quad \cancel{\underline{\quad}} \quad \alpha + \sqrt{2}\beta$$

$$\text{Comparing to } 8(\alpha + \beta), \\ \text{delocalization energy is } 4(\sqrt{2}-1)\beta \approx 1.64\beta$$

$$\text{HOMO} \quad \cancel{\underline{\quad}} \quad \alpha + 2\beta$$