Chem. 540
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Hydrogen Atom Problem 5

Consider two particles described by the 3-dimensional coordinate vectors $\mathbf{r}_1, \mathbf{r}_2$ in an arbitrary Cartesian coordinate system. Define the center-of-mass and relative coordinate vectors

$$\mathbf{R} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{M}, \quad \mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$$

where $M = m_1 + m_2$ is the total mass. Show that the classical kinetic energy of the two-particle system,

$$T = \frac{1}{2} m_1 |\dot{\mathbf{r}}_1|^2 + \frac{1}{2} m_2 |\dot{\mathbf{r}}_2|^2$$

(where the dot denotes the time derivative) takes the form

$$T = \frac{1}{2} M |\dot{\mathbf{R}}|^2 + \frac{1}{2} \mu |\dot{\mathbf{r}}|^2,$$

where

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

is the reduced mass. Thus, assuming that the interaction between the two particles depends only on the distance between them, conclude that the problem separates into a free-particle translational motion for the center-of-mass coordinate and the internal coordinate motion described by the Hamiltonian

$$H = \frac{|\mathbf{p}|^2}{2\mu} + V(r)$$

Where $\mathbf{p}$ is the momentum vector conjugate to the internal coordinate $\mathbf{r}$, i.e., $\mathbf{p} = \mu\dot{\mathbf{r}}$.

Finally, calculate the reduced mass for a nucleus+electron system (i.e., a hydrogen atom).