Chem. 540

Hydrogen Problem 3 - Solution

$$\langle r \rangle = \int_0^{\infty} dr \int_0^{\pi} d\theta \int_0^{2\pi} d\varphi \left[R_{10}(r)\, Y_{00}(\theta, \varphi) \right]^2 r^2 \sin\theta \cdot r$$

Because $\int_0^{\pi} d\theta \int_0^{2\pi} d\varphi \, |Y_{lm}(\theta, \varphi)|^2 \sin\theta = 1$,

$$\langle r \rangle = \int_0^{\infty} dr \; r^3 \, R_{10}(r)^2 = \int_0^{\infty} dr \; r^3 \left(2 a_0^{-3/2} e^{-r/a_0} \right)^2 = \frac{3}{2} a_0$$

where $a_0 = \dfrac{4\pi \varepsilon_0 \hbar^2}{m_e e^2}$ is the Bohr radius.