$$\dot{\vec{R}} = \frac{m}{M}\dot{\vec{r}}_1 + \frac{m_2}{M}\dot{\vec{r}}_2 , \qquad \dot{\vec{r}} = \dot{\vec{r}}_2 - \dot{\vec{r}}_1 \qquad |\dot{\vec{r}}|^2 = |\dot{\vec{r}}_2|^2 + |\dot{\vec{r}}_1|^2 - 2\dot{\vec{r}}_1\cdot\dot{\vec{r}}_2$$

$$|\dot{\vec{R}}|^2 = \left(\frac{m_1}{M}\dot{\vec{r}}_1 + \frac{m_2}{M}\dot{\vec{r}}_2\right)\cdot\left(\frac{m_1}{M}\dot{\vec{r}}_1 + \frac{m_2}{M}\dot{\vec{r}}_2\right) = \left(\frac{m_1}{M}\right)^2|\dot{\vec{r}}_1|^2 + \left(\frac{m_2}{M}\right)^2|\dot{\vec{r}}_2|^2 + \frac{2m_1m_2}{M^2}\dot{\vec{r}}_1\cdot\dot{\vec{r}}_2$$

$$\frac{1}{2}M|\dot{\vec{R}}|^2 + \frac{1}{2}\mu|\dot{\vec{r}}|^2 = \frac{1}{2M}\cdot\left(m_1^2|\dot{\vec{r}}_1|^2 + m_2^2|\dot{\vec{r}}_2|^2 + 2m_1m_2\dot{\vec{r}}_1\cdot\dot{\vec{r}}_2\right)$$

$$+ \frac{1}{2}\frac{m_1m_2}{M}\left(|\dot{\vec{r}}_2|^2 + |\dot{\vec{r}}_1|^2 - 2\dot{\vec{r}}_1\cdot\dot{\vec{r}}_2\right)$$

$$= \frac{m_1}{2M}(m_1+m_2)|\dot{\vec{r}}_1|^2 + \frac{m_2}{2M}(m_1+m_2)|\dot{\vec{r}}_2|^2 = \frac{1}{2}m_1|\dot{\vec{r}}_1|^2 + \frac{1}{2}m_2|\dot{\vec{r}}_2|^2$$

So $$H = \frac{1}{2}m_1\dot{\vec{r}}_1^{\,2} + \frac{1}{2}m_2\dot{\vec{r}}_2^{\,2} + V(|\vec{r}_2-\vec{r}_1|) = \underbrace{\frac{1}{2}M|\dot{\vec{R}}|^2}_{\text{translation}} + \underbrace{\frac{1}{2}\mu|\dot{\vec{r}}|^2 + V(r)}_{\text{internal motion } H_{int}}.$$

$$H_{int} = \frac{|\vec{p}|^2}{2\mu} + V(r), \qquad \vec{p} = \mu\dot{\vec{r}}.$$

If $m_1 = 1.67262\times10^{-27}$ kg (proton), $m_2 = 9.10938\times10^{-31}$ kg (electron),

$\mu = 9.10442\times10^{-31}$ kg