

$$R_{1s}(r) = 2a^{-3/2} e^{-r/a}$$

$$\langle V \rangle = \int_0^{\infty} 4a^{-3} e^{-2r/a} \left(\frac{-e^2}{4\pi\epsilon_0 r} \right) r^2 dr \quad a = \frac{4\pi\epsilon_0 \hbar^2}{\mu e^2}$$

$$\text{so } \frac{e^2}{4\pi\epsilon_0} = \frac{\hbar^2}{a\mu}$$

$$\langle V \rangle = -4a^{-3} \frac{\hbar^2}{a\mu} \int_0^{\infty} r e^{-2r/a} dr = -\frac{4\hbar^2}{a^4\mu} \cdot \frac{1}{(2/a)^2} = -\frac{\hbar^2}{a^2\mu}$$

$$\langle V \rangle = -\frac{\hbar^2}{\mu} \left(\frac{\mu e^2}{4\pi\epsilon_0 \hbar^2} \right)^2 = -\frac{\mu e^4}{16\pi^2 \epsilon_0^2 \hbar^2} = 2 E_{1s}$$

$$\left\langle \frac{\hat{p}^2}{2m} \right\rangle = E_{1s} - \langle V \rangle = -E_{1s}$$