

1. Expand in Taylor Series about $x=0$ up to quadratic terms

$$a) e^x \approx e^0 + \frac{x-0}{1!} e^0 + \frac{(x-0)^2}{2!} e^0$$

$$e^x \approx 1 + x + \frac{1}{2}x^2$$

$$b) xe^{-x^2} \approx 0 + \frac{x}{1!} e^0(1-0) + \frac{x^2}{2!} (0)$$

$$xe^{-x^2} \approx x$$

$$c) \cos x \approx \cos 0 + x(-\sin 0) + \frac{x^2}{2}(-\cos 0)$$

$$\cos x \approx 1 - \frac{x^2}{2}$$

$$d) \ln(1+x) \approx \ln(1) + \frac{x}{1} \left(\frac{1}{1+0}\right) + \frac{x^2}{2} \frac{-1}{(1+0)^2}$$

$$\ln(1+x) \approx x - \frac{1}{2}x^2$$

$$e) \sqrt{1+x^2} \approx \sqrt{1} + \frac{x}{1} \frac{0}{\sqrt{1+0}} + \frac{x^2}{2} \left(\frac{1}{\sqrt{1+0}} + \frac{0}{(1+0)^{3/2}} \right)$$

$$\sqrt{1+x^2} \approx 1 + \frac{1}{2}x^2$$

f) $x^4 - x^2$: this is already in series form, but expanding to quadratic order:

$$x^4 - x^2 \approx (0^4 - 0^2) + \frac{x}{1} (4 \cdot 0^3 - 2 \cdot 0) + \frac{x^2}{2} (12(0)^2 - 2)$$

$$x^4 - x^2 \approx -x^2$$