

$$3. a) f = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4$$

$$f' = -ax + bx^3 = 0$$

$$x = 0, \pm\sqrt{\frac{a}{b}}$$

$$f'' = -a + 3bx^2$$

$$f''(0) = -a \quad f''(\sqrt{\frac{a}{b}}) = -a + 3b(\frac{a}{b}) = 2a \quad f''(-\sqrt{\frac{a}{b}}) = 2a$$

If $a < 0$: Maxima at $\pm\sqrt{\frac{a}{b}}$, minimum at 0

If $a > 0$: Maximum at $x=0$, minima at $\pm\sqrt{\frac{a}{b}}$

$$b) f = x^2 e^{-x^2}$$

$$f' = e^{-x^2} (2x)(1-x^2) = 0$$

$$x = 0, \pm 1$$

$$f'' = e^{-x^2} (2 - 10x^2 + 4x^4)$$

$$f''(0) = e^0 (2 - 0 - 0) = 2$$

$$f''(1) = e^{-1} (2 - 10 + 4) = -4/e$$

$$f''(-1) = -4/e$$

Minimum at $x=0$, maxima at $x=\pm 1$