4. a) $\int_0^\infty e^{-ax}dx = -\frac{1}{a}e^{-ax}\Big|_0^\infty = -\frac{1}{a}(e^{-\infty}-e^0) = \frac{1}{a}$

b) $\int_0^\infty xe^{-ax}dx = x\left(-\frac{1}{a}e^{-ax}\right) - \int -\frac{1}{a}e^{-ax}dx$

$= \left[-\frac{x}{a}e^{-ax} + \frac{1}{a^2}e^{-ax}\right]\Big|_0^\infty = \frac{1}{a^2}$

c) $\int_{-\infty}^\infty xe^{-ax^2}dx = -\frac{1}{2a}e^{-u}\Big|_\infty^\infty = 0$

d) $\int_0^\infty xe^{-ax^2}dx = -\frac{1}{2a}e^{-u}\Big|_0^\infty = -\frac{1}{2a}(0-1) = \frac{1}{2a}$

e) $\int_{-\infty}^\infty e^{-ax^2+bx}dx = e^{ac^2}\int_{-\infty}^\infty e^{-a(x-c)^2}dx$ (where $c = b/2a$)

$= e^{b^2/4a}\int_{-\infty}^\infty e^{-a(x-b/2a)^2}dx$

$= e^{b^2/4a}\sqrt{\frac{\pi}{a}}$