

Chem. 540

Math Problem 8 - Solution

$$\vec{s} = \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} \quad \vec{v} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} \quad \vec{w} = \begin{pmatrix} -1 \\ 8 \\ a \end{pmatrix}$$

$$(a) \quad \vec{s}^T \cdot \vec{s} = (4 \ -2 \ 3) \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} = 16 + 4 + 9 = 29 \Rightarrow |\vec{s}| = \sqrt{29}$$

$$\vec{v}^T \cdot \vec{v} = (1 \ 3 \ -1) \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} = 1 + 9 + 1 = 11 \Rightarrow |\vec{v}| = \sqrt{11}$$

$$(b) \quad \vec{s}^T \cdot \vec{v} = (4 \ -2 \ 3) \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} = 4 - 6 - 3 = -5$$

$$\vec{v}^T \cdot \vec{s} = (1 \ 3 \ -1) \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} = 4 - 6 - 3 = -5$$

$$\vec{s}^T \cdot \vec{w} = (4 \ -2 \ 3) \begin{pmatrix} -1 \\ 8 \\ a \end{pmatrix} = -4 - 16 + 3a = 3a - 20$$

$$\vec{w}^T \cdot \vec{s} = (-1 \ 8 \ a) \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} = -4 - 16 + 3a = 3a - 20$$

$$(c) \quad \text{Want } 3a - 20 = 0 \Rightarrow a = \frac{20}{3}$$

$$(d) \quad \vec{s} \cdot \vec{v}^T = \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} (1 \ 3 \ -1) = \begin{pmatrix} 4 & 12 & -4 \\ -2 & -6 & 2 \\ 3 & 9 & -3 \end{pmatrix}, \quad \vec{v} \cdot \vec{s}^T = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} (4 \ -2 \ 3) = \begin{pmatrix} 4 & -2 & 3 \\ 12 & -6 & 9 \\ -4 & 2 & -3 \end{pmatrix}$$