

## Math Problem 9 - Solution

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 2 & -1 & 0 \\ 0 & 5 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 0 & -2 \\ 4 & 1 & 1 \\ 1 & 3 & 2 \end{pmatrix}$$

$$(a) \quad A \cdot B = \begin{pmatrix} 15 & 6 & 3 \\ 0 & -1 & -5 \\ 22 & 11 & 9 \end{pmatrix} \quad B \cdot A = \begin{pmatrix} 2 & -4 & -2 \\ 6 & 16 & 6 \\ 7 & 10 & 5 \end{pmatrix}$$

$$(b) \quad \det A = 1 \cdot \begin{vmatrix} -1 & 0 \\ 5 & 2 \end{vmatrix} - 2 \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = -2 - 2 \cdot 1 = -4$$

$$\det B = 2 \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} - 2 \begin{vmatrix} 4 & 1 \\ 1 & 3 \end{vmatrix} = 2 \cdot (-1) - 2 \cdot 11 = -24$$

$$\det A \cdot B = 15 \begin{vmatrix} -1 & -5 \\ 11 & 9 \end{vmatrix} + 22 \begin{vmatrix} 6 & 3 \\ -1 & -5 \end{vmatrix} = 15 \cdot 46 + 22 \cdot (-27) = 96$$

$$\begin{aligned} \det B \cdot A &= 2 \begin{vmatrix} 16 & 6 \\ 10 & 5 \end{vmatrix} - 6 \begin{vmatrix} -4 & -2 \\ 10 & 5 \end{vmatrix} + 7 \begin{vmatrix} -4 & -2 \\ 16 & 6 \end{vmatrix} \\ &= 2 \cdot 20 - 6 \cdot 0 + 7 \cdot 8 = 96 \end{aligned}$$

Notice  $\det(A \cdot B) = \det(B \cdot A) = \det A \cdot \det B$ .

$$(c) \quad A^T = \begin{pmatrix} 1 & 2 & 0 \\ 3 & -1 & 5 \\ 1 & 0 & 2 \end{pmatrix} \quad B^T = \begin{pmatrix} 2 & 4 & 1 \\ 0 & 1 & 3 \\ -2 & 1 & 2 \end{pmatrix}$$

$$\det A^T = 1 \cdot (-2) - 2 \cdot 1 = -4 = \det A, \quad \det B^T = 2 \cdot (-1) - 2 \cdot 11 = -24 = \det B$$

$$\begin{aligned} A^T \cdot B &= \begin{pmatrix} 1 & 2 & 0 \\ 3 & -1 & 5 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & 0 & -2 \\ 4 & 1 & 1 \\ 1 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 10 & 2 & 0 \\ 7 & 14 & 3 \\ 4 & 6 & 2 \end{pmatrix} \quad \det A^T \cdot B = 10 \cdot 10 - 2 \cdot 2 = 96 \\ &= \det A^T \det B = \det A \det B. \end{aligned}$$