Chem. 540
Instructor: Nancy Makri

Models – Problem 2

The uncertainty Δx in the position of a particle is given by the root mean square deviation of the particle from its mean position; i.e.,

$$\Delta x = \left(\left\langle x^2 \right\rangle - \left\langle x \right\rangle^2 \right)^{\frac{1}{2}}.$$

Similarly, the uncertainty Δp in the momentum of a particle is given by the root mean square deviation of the particle from its mean momentum:

$$\Delta p = \left(\left\langle p^2 \right\rangle - \left\langle p \right\rangle^2 \right)^{\frac{1}{2}}$$

Consider again a particle in a one-dimensional square well of length L.

a) Calculate $\left\langle x \right\rangle$ and $\left\langle x^2 \right\rangle$ for this system. Use your results to find the uncertainty Δx in position.

b) Calculate $\left\langle p \right\rangle$ and $\left\langle p^2 \right\rangle$ for this system by using the form of the momentum operator in position space. Notice that there is another way to obtain $\left\langle p^2 \right\rangle$ in this case, using the relation between the square of the momentum and the total energy (which is known for every value of n). Use this observation to check your result.

c) Use your results to find the uncertainty Δp in momentum and show that $\Delta x \Delta p \geq \hbar / 2$ for all values of n, consistent with the uncertainty principle.