

2b. Consider a particle in a one dimensional square well of length L with infinite walls at $x=0$ and $x=L$.

a) Calculate probability of finding particle between $x-\lambda/2$ and $x+\lambda/2$.

$$P(x; \lambda) = \int_{x-\lambda/2}^{x+\lambda/2} dx \left(\frac{2}{L}\right) \sin^2\left(\frac{n\pi x}{L}\right) = \frac{2}{L} \int_{x-\lambda/2}^{x+\lambda/2} dx \left(\frac{1 - \cos\left(\frac{2n\pi x}{L}\right)}{2}\right)$$

$$P(x; \lambda) = \frac{x}{L} \Big|_{x-\lambda/2}^{x+\lambda/2} - \frac{2}{L} \left(\frac{1}{2}\right) \left(\frac{L}{2n\pi}\right) \sin\left(\frac{2n\pi x}{L}\right) \Big|_{x-\lambda/2}^{x+\lambda/2}$$

$$P(x; \lambda) = \frac{\lambda}{L} - \frac{1}{2n\pi} \left[\sin\left(\frac{2n\pi(x+\lambda/2)}{L}\right) - \sin\left(\frac{2n\pi(x-\lambda/2)}{L}\right) \right]$$

b) For a classical probability; there is no preferred location for the particle. The total probability of finding the particle in the well is 1 and the portion of the well between $x-\lambda/2$ and $x+\lambda/2$ is λ/L of the entire well. The probability is $1 \cdot \frac{\lambda}{L} = \frac{\lambda}{L}$.

c). As $n \rightarrow \infty$:

$$\lim_{n \rightarrow \infty} \frac{\lambda}{L} - \frac{1}{2n\pi} \left[\sin\left(\frac{2n\pi(x+\lambda/2)}{L}\right) - \sin\left(\frac{2n\pi(x-\lambda/2)}{L}\right) \right] = \frac{\lambda}{L}$$

As the quantum number becomes large, we recover the classical limit.