$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}} \left(\hat{a} + \hat{a}^{+}\right)$$

$$\langle i | \hat{x} | j \rangle = \sqrt{\frac{\hbar}{2m\omega}} \langle i | \hat{a} + \hat{a}^{+}) | j \rangle$$

$$= \sqrt{\frac{\hbar}{2m\omega}} \left( \sqrt{j} \langle i | j-1 \rangle + \sqrt{j+1} \langle i | j+1 \rangle \right)$$

$$= \sqrt{\frac{\hbar}{2m\omega}} \left( \sqrt{j}\, \delta_{i,j-1} + \sqrt{j+1}\, \delta_{i,j+1} \right)$$

$$\langle \hat{x}^2 \rangle = \langle \phi_n | \hat{x}^2 | \phi_n \rangle = \sum_m \langle \phi_n | \hat{x} | \phi_m \rangle \langle \phi_m | \hat{x} | \phi_n \rangle$$

$$= \frac{\hbar}{2m\omega} \sum_m \left( \sqrt{m}\, \delta_{n,m-1} + \sqrt{m+1}\, \delta_{n,m+1} \right)^2$$

$$= \frac{\hbar}{2m\omega} \Big[ \underbrace{\left( \sqrt{n+1} + 0 \right)^2}_{m=n+1} + \underbrace{\left( 0 + \sqrt{n} \right)^2}_{m=n-1} \Big] = \frac{\hbar}{2m\omega} \left( 2n+1 \right).$$