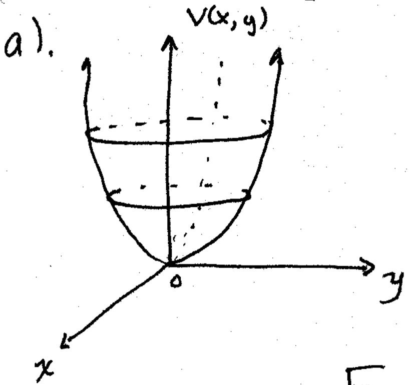
31. $\hat{H} = \frac{\hat{p}_x^2}{2m} + \frac{\hat{p}_y^2}{2m} + V(\hat{x},\hat{y}) \quad V(\hat{x},\hat{y}) = \frac{1}{2}m\omega_x^2\hat{x}^2 + \frac{1}{2}m\omega_y^2\hat{y}^2$

a).




$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \frac{\hat{p}_y^2}{2m} + \frac{1}{2}m\omega_x^2\hat{x}^2 + \frac{1}{2}m\omega_y^2\hat{y}^2$$

$$\hat{H} = \underbrace{\frac{\hat{p}_x^2}{2m} + \frac{1}{2}m\omega_x^2\hat{x}^2}_{\hat{H}_x} + \underbrace{\frac{\hat{p}_y^2}{2m} + \frac{1}{2}m\omega_y^2\hat{y}^2}_{\hat{H}_y} = \hat{H}_x + \hat{H}_y$$

Since it is the sum of two uncoupled harmonic oscillators, the eigenvalues are the sum of the two independent harmonic oscillators.

$$E_{n_x n_y} = \hbar\omega_x(n_x + \tfrac{1}{2}) + \tfrac{1}{2}\omega_y(n_y + \tfrac{1}{2})$$

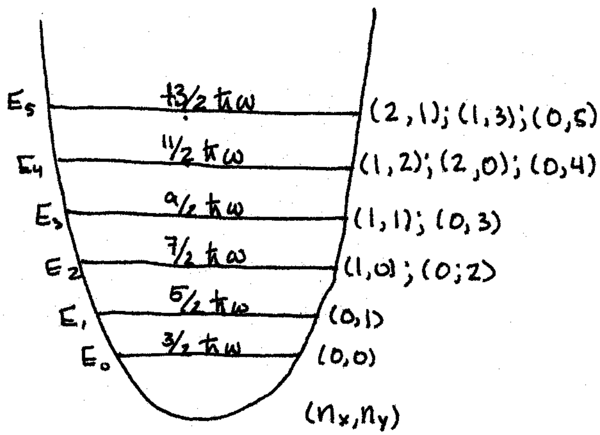
If $\omega_x = \omega_y$, there are degeneracies. Degeneracies can occur when $\omega_x/\omega_y$ is an integer.

b) Suppose $\omega_x = 2\omega_y$:

$$E_{n_x n_y} = 2\hbar\omega(n_x + \tfrac{1}{2}) + \hbar\omega(n_y + \tfrac{1}{2})$$

$$E_{n_x n_y} = \hbar\omega(2n_x + n_y + \tfrac{3}{2})$$

Lowest six energy levels:




| $n_x$ | $n_y$ | $E$ |
|---|---|---|
| 0 | 0 | $\frac{3}{2}\hbar\omega$ |
| 0 | 1 | $\frac{5}{2}\hbar\omega$ |
| 1 | 0 | $\frac{7}{2}\hbar\omega$ |
| 0 | 2 | $\frac{7}{2}\hbar\omega$ |
| 1 | 1 | $\frac{9}{2}\hbar\omega$ |
| 0 | 3 | $\frac{9}{2}\hbar\omega$ |
| 1 | 2 | $\frac{11}{2}\hbar\omega$ |
| 2 | 0 | $\frac{11}{2}\hbar\omega$ |
| 0 | 4 | $\frac{11}{2}\hbar\omega$ |
| 2 | 1 | $\frac{13}{2}\hbar\omega$ |
| 1 | 3 | $\frac{13}{2}\hbar\omega$ |
| 0 | 5 | $\frac{13}{2}\hbar\omega$ |