$\psi_1(x) = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}$ $\qquad\qquad$ $P_1(x) = \psi_1(x)^2$.

Maximum P :

$$\frac{d}{dx} P_n(x) = \frac{d}{dx} \psi_n(x)^2 = 2\,\psi_n(x)\,\psi_n'(x)$$

For maximum, $\psi_n(x) \neq 0$ so $\psi_n'(x) = 0$, ie we can look for extrema of ψ_n.

$$\psi_1'(x) = \sqrt{\frac{2}{L}}\,\frac{\pi}{L} \cos\frac{\pi x}{L} = 0 \quad \text{at} \quad \frac{\pi x}{L} = \frac{\pi}{2} \quad \text{ie} \quad x = \frac{L}{2}.$$

$\psi_1\left(\frac{L}{2}\right) = \sqrt{\frac{2}{L}}$ and $P_1\left(\frac{L}{2}\right) = \frac{2}{L}$.

At which x is $P_1(x) = \frac{1}{2} \cdot \frac{2}{L}$?

$\frac{2}{L} \sin^2 \frac{\pi x}{L} = \frac{1}{2} \cdot \frac{2}{L}$, $\quad \sin^2 \frac{\pi x}{L} = \frac{1}{2}$, $\quad \sin \frac{\pi x}{L} = \frac{\sqrt{2}}{2}$ so

$\frac{\pi x}{L} = \frac{\pi}{4}$ or $\frac{3\pi}{4}$, $\quad x = \frac{1}{4} L$ or $\frac{3}{4} L$.