

$$\begin{aligned}
 (a) \quad \langle 0 | \hat{x}^2 | 2 \rangle &= \frac{\hbar}{2m\omega} \left(\langle 0 | \hat{a} \hat{a} | 2 \rangle + \langle 0 | \hat{a}^{\dagger} \hat{a} | 2 \rangle + \langle 0 | \hat{a} \hat{a}^{\dagger} | 2 \rangle + \langle 0 | \hat{a}^{\dagger} \hat{a}^{\dagger} | 2 \rangle \right) \\
 &= \frac{\hbar}{2m\omega} \left(\sqrt{2}\sqrt{1} \langle 0 | 0 \rangle + \sqrt{2}\sqrt{2} \langle 0 | 2 \rangle + \sqrt{3}\sqrt{3} \langle 0 | 2 \rangle + \sqrt{3}\sqrt{4} \langle 0 | 4 \rangle \right) \\
 &= \frac{1}{\sqrt{2}} \frac{\hbar}{m\omega}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad \langle 0 | \hat{x}^2 | 1 \rangle &= \frac{\hbar}{2m\omega} \left(\langle 0 | \hat{a} \hat{a} | 1 \rangle + \langle 0 | \hat{a}^{\dagger} \hat{a} | 1 \rangle + \langle 0 | \hat{a} \hat{a}^{\dagger} | 1 \rangle + \langle 0 | \hat{a}^{\dagger} \hat{a}^{\dagger} | 1 \rangle \right) \\
 &= 0 \quad \begin{array}{l} \downarrow \\ \propto |0\rangle, \\ \langle 0 | \hat{a} | 0 \rangle = 0 \end{array}
 \end{aligned}$$

(Could have guessed based on symmetry.)

$$\begin{aligned}
 (c) \quad \langle 1 | \hat{x}^3 | 4 \rangle &= \left(\frac{\hbar}{2m\omega} \right)^{3/2} \left(\langle 1 | \hat{a} \hat{a} \hat{a} | 4 \rangle + \dots \quad (\text{all give zero}) \right) \\
 &= \left(\frac{\hbar}{2m\omega} \right)^{3/2} \left(\sqrt{4}\sqrt{3}\sqrt{2} \langle 1 | 1 \rangle \right) = \sqrt{3} \left(\frac{\hbar}{m\omega} \right)^{3/2}.
 \end{aligned}$$

$$(d) \quad \langle n | \hat{x}^2 | n-1 \rangle = \int dx \phi_n(x) x^2 \phi_{n-1}(x) = 0 \quad \text{by symmetry,}$$

since the product $\phi_n \phi_{n-1}$ is an odd function

$$\begin{aligned}
 (e) \quad \langle n | \hat{x}^2 | n+2 \rangle &= \frac{\hbar}{2m\omega} \left(\langle n | \hat{a} \hat{a} | n+2 \rangle + \dots \quad (\text{all zero}) \right) \\
 &= \frac{\hbar}{2m\omega} \sqrt{n+2} \sqrt{n+1}
 \end{aligned}$$