

36. Particle in a 1-D box. Add perturbation $V(\hat{x}) = c\hat{x}$.

$$H = H_0 + V(\hat{x}) = \frac{\hat{p}^2}{2m} + c\hat{x}$$

$$H_0\phi_n = \epsilon_n\phi_n, \text{ where } \phi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \quad n=1, 2, \dots$$

$$\epsilon_n = \frac{\hbar^2}{2m} \left(\frac{n\pi}{L}\right)^2 \quad n=1, 2, \dots$$

Non-degenerate perturbation:

$$E_n^{(1)} = \langle \phi_n | \hat{V} | \phi_n \rangle \quad E_n^{(2)} = \langle \phi_n | \hat{V} \frac{\hat{Q}_n}{\epsilon_n - \hat{H}_0} \hat{V} | \phi_n \rangle$$

$$E_n^{(1)} = \int_0^L \left(\frac{2}{L}\right) \sin^2\left(\frac{n\pi x}{L}\right) c x dx = \frac{2c}{L} \int_0^L x \sin^2\left(\frac{n\pi x}{L}\right) dx$$

$$E_n^{(1)} = \frac{2c}{L} \left(\frac{L^2}{4}\right) = \frac{cL}{2}$$

Energy through 1st order in perturbation:

$$E = \epsilon_n + \frac{cL}{2}$$