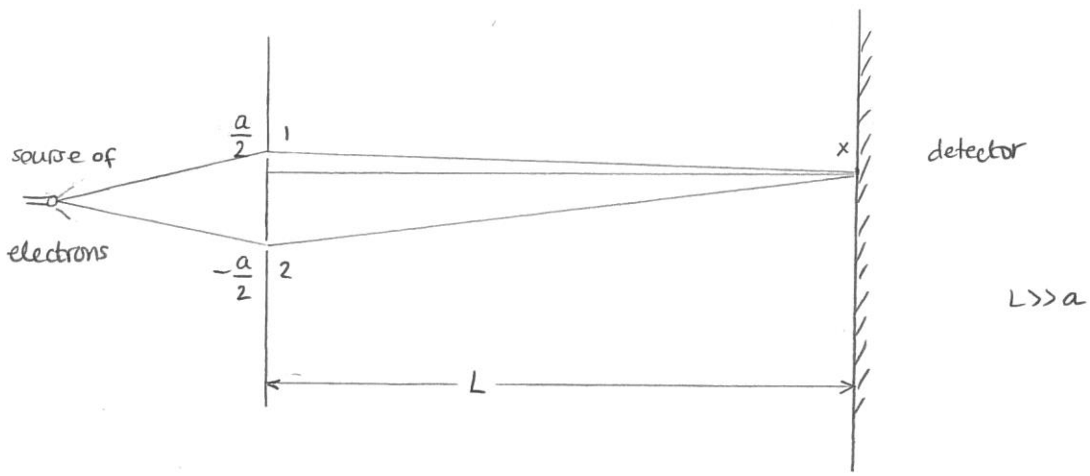
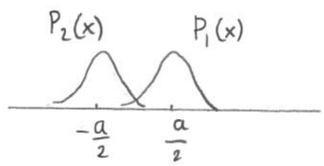
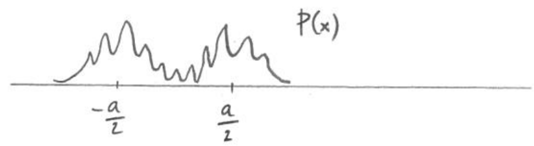
The two-slit experiment

Discussion of the experiment with microscopic particles ("electrons"):





Observed distributions if only one slit is open



Observed distribution when both slits are open

Explanation: An electron is a wave. The probability amplitude of arriving at x if it went through slit 1 is $S_1(x)$. The probability is the square modulus of the amplitude, so

$$P_1(x) = |S_1(x)|^2, \qquad P_2(x) = |S_2(x)|^2 .$$

The total amplitude is the sum of individual amplitudes, so $S(x) = S_1(x) + S_2(x)$.

From wave mechanics and since $|S_1(x)|^2$ must be bell-shaped,

$$S_1(x) = |S_1(x)|\, e^{2\pi i l_1/\lambda_{el}} \qquad S_2(x) = |S_2(x)|\, e^{2\pi i l_2/\lambda_{el}}$$

where $\lambda_{el} = \dfrac{h}{p_{el}}$ de Broglie relation (h = Planck's constant)

So $P(x) = |S_1(x)|^2 + |S_2(x)|^2 + 2\,\mathrm{Re}\, S_1(x)^* S_2(x)$

$$= P_1(x) + P_2(x) + \underbrace{2|S_1(x)||S_2(x)| \cos \frac{2\pi \Delta l}{\lambda_{el}}}_{\text{interference term!}}, \qquad \Delta l \equiv |l_1 - l_2|$$

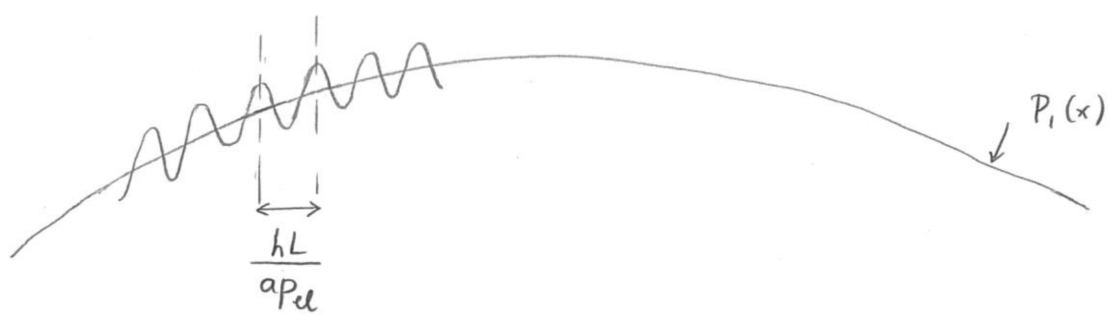
Let's simplify this, assuming $L >> a$: from right triangles,

$$\Delta l = \sqrt{L^2 + \left(x + \frac{a}{2}\right)^2} - \sqrt{L^2 + \left(x - \frac{a}{2}\right)^2} = L\left[1 + \left(\frac{x + \frac{a}{2}}{L}\right)^2\right]^{1/2} - L\left[1 + \left(\frac{x - \frac{a}{2}}{L}\right)^2\right]^{1/2}$$

$$\simeq L\left[1 + \frac{1}{2}\frac{\left(x + \frac{a}{2}\right)^2}{L^2}\right] - L\left[1 + \frac{1}{2}\frac{\left(x - \frac{a}{2}\right)^2}{L^2}\right] = \frac{ax}{L}$$

So the interference term is $2|S_1(x)||S_2(x)| \cos \frac{2\pi a x}{L\lambda_{el}}$. Using de Broglie, this becomes

$$2|S_1(x)||S_2(x)| \cos 2\pi \frac{a p_{el}}{hL} x$$



Now suppose we try to watch the electrons, in an attempt to understand this weird interference effect. We place a light source behind the slits so we are able to see whether each electron went through hole 1 or 2. Surprisingly, the interference is now lost, i.e. the electrons behave just like classical particles.

Explanation: Light is made up of photons, which carry momentum

$$p_{ph} = \frac{h}{\lambda_{ph}} .$$

The scattering of light is a photon-electron collision, which changes the position of an electron by the amount

$$\Delta x \simeq \frac{\Delta p_{el}}{m_{el}} t \simeq \frac{\Delta p_{ph}}{m_{el}} \cdot t \simeq \frac{p_{ph}}{m_{el}} \cdot \frac{L}{(p_{el}/m_{el})} = \frac{h}{\lambda_{ph}} \cdot \frac{L}{p_{el}}$$

To be able to see whether an electron went through slit 1 or through 2, we need to use light with

$$\lambda_{ph} < a .$$

But then $\Delta x > \frac{hL}{a p_{el}}$

So the positions of the electrons are perturbed by amounts larger than the fine scale of the interference pattern. As a result, this destroys the interference pattern.